On the Nature of Spin Currents

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Full expressions for finite frequency spin, charge conductivity and spin susceptibility in Rashba and Luttinger-type systems are given. Whereas in the Rashba Hamiltonian the spin conductivity has the same frequency dependence as the dielectric polarizability [1] and magnetic susceptibility [2, 3], the Luttinger case is different. Moreover, for a generalized Rashba-type coupling the three quantities also exhibit different frequency dependencies.

PACS numbers: 72.25.-b, 72.10.-d, 72.15. Gd

In this short note we provide the full frequency dependence of spin conductivity, dielectric function, and magnetic susceptibility in Rashba and Luttinger-type systems. In general, for the Luttinger case or for a generalized Rashba-type spin-orbit coupling, the frequency dependencies are different, but for the two dimensional Rashba Hamiltonian they become the same, as noticed before [1, 2, 3]. However, this seems to be the exception rather than the rule.

Recent theoretical work predicts the existence of spin currents in semiconductors with spin-orbit coupling placed under the influence of an electric field [4, 5]. In one of the proposals [4], spin current is induced in p-doped cubic bulk semiconductors with Luttinger-type spin-orbit coupling of the spin-3/2 valence band when an electric field is applied to the material. In the other proposal [5] spin current is induced in an n-doped 2-dimensional semiconductor layer upon the application of an in-plane electric field. In both cases, the electric field, the spin polarization and the spin direction of flow are mutually perpendicular. The spin currents are 'dissiplationless' in the sense that they do not depend on the momentum scattering rate, as normal charge current does. The presence of a charge current due to the applied electric field does however cause dissipation.

Apart from the experimental proof of their existence (the idea of an experimental spin-voltmeter is yet unimaginable, but experimental detection of spin accumulation due to spin current at the boundaries of a sample has recently been reported [6]), many other fundamental questions on the nature of spin currents remain. In a recent paper [1], Rashba focuses on the essential physical question of whether spin current is a stand-alone phenomena or is it described macroscopically through the spin-orbit contribution to the dielectric function. He finds that for the case of the Rashba Hamiltonian the frequency dependence of the spin conductance and dielectric function is the same, strongly pointing to the fact that the two are indeed the same effect. Also, Erlingsson *et al* and Dimitrova relate the spin current in the Rashba model to the magnetic susceptibility, linking it to the uniform spin polarization that also develops in the presence of an applied electric field. As we show below, this is not the case in the Luttinger Hamiltonian, nor is it the case for a generalized Rashba-type coupling.

Let us start with a generalized conduction band Rashba-type spin-orbit Hamiltonian:

$$H_R = \varepsilon(k) + \lambda_i(\mathbf{k})\sigma_i, \quad \varepsilon(k) = \frac{k^2}{2m_e}, \quad i = 1,2,3 \qquad (1)$$

where m_e is the effective mass in the conduction band, σ_i are the Pauli matrices and $\lambda_i(\mathbf{k})$ are generic functions of the momentum $\mathbf{k}$ which respect the symmetry of the underlying crystal (the Rashba Hamiltonian is given by the particular case of a 2 dimensional k space and by $\lambda_x = \alpha k_y$, $\lambda_y = -\alpha k_x$, $\lambda_3 = 0$ but in general we can have all three λ_i non-zero such as in the bulk Dresselhaus splitting.) The energy eigenvalues are $E_\pm = \varepsilon(k) \pm \lambda(k)$ where $\lambda(k) = \sqrt{\lambda^i\lambda^i}$. The Green's function reads:

$$G(\mathbf{k}, i\omega) = \frac{1}{i\omega + \mu - H_R} = \frac{i\omega + \mu - \varepsilon(k) + \lambda_i(k)\sigma_i}{(i\omega + \mu - \varepsilon(k))^2 - \lambda^2(k)} \qquad (2)$$

and the charge current operator is $J_i = \frac{\partial H_R}{\partial k_i}$ where $i = 1, 2, 3$. The spin-orbit induced part of the polarisability tensor ϵ_{ij} is related to the charge conductivity tensor σ_{ij} and to the response function Q_{ij} in the following way:

$$\epsilon_{ij}(\omega) = \frac{4\pi i \sigma_{ij}(\omega)}{\omega}$$

$$\sigma_{ij}(\omega) = \frac{e^2}{2\hbar}\frac{Q_{ij}(\omega)}{-i\omega}$$

$$Q_{ij}(i\nu_m) = -\frac{1}{V}\int_0^\beta \langle T J_i(u) J_j\rangle e^{i\nu_m u} du =$$
$$= \frac{1}{V\beta}\sum_{k,n} \mathrm{tr}(J_i G(\mathbf{k}, i(\omega_n + \nu_m)) J_j G(\mathbf{k}, i\omega_n)) =$$
$$= -\frac{2}{V}\sum_k \frac{n^F(E_+) - n^F(E_-)}{\lambda[(i\nu_m)^2 - (2\lambda)^2]} \times$$
$$\times [\nu_m \epsilon_{abc}\lambda_b \frac{\partial\lambda_a}{\partial k_i}\frac{\partial\lambda_c}{\partial k_j} + 2\lambda^2(\frac{\partial\lambda}{\partial k_i}\frac{\partial\lambda}{\partial k_j} - \frac{\partial\lambda_a}{\partial k_i}\frac{\partial\lambda_a}{\partial k_j})] \qquad (3)$$

where $\nu_m = 2\pi/\beta$, n^F represents the Fermi function, ϵ_{abc} is the totally antisymmetric tensor and we sum on any repeated index. As an essential observation, the current response function of Eq.[3] does **not** depend on the kinetic energy $\varepsilon(k)$ (except through the Fermi functions) and is entirely given by the spin-orbit coupling terms λ_i.

Let us now compute the spin-current charge-current correlation function Q^l_{ij} which gives the response of the spin current J^l_i to an applied electric field E_j. The spin current operator for spin flowing in the i direction polarized in the l direction is $J^l_i = \frac{1}{2}\{\frac{\partial H}{\partial k_i}, \sigma^l\}$. The spin current in this system is **not** a conserved quantity. The spin conductivity σ^l_{ij} and the spin current-charge current correlation function read Q^l_{ij} read:

$$\sigma^l_{ij} = e\frac{Q^l_{ij}(\omega)}{-i\omega}$$

$$\begin{aligned} Q^l_{ij}(i\nu_m) &= -\frac{1}{V}\int_0^\beta \langle T J^l_i(u) J_j\rangle e^{i\nu_m u} du = \\ &= \frac{1}{V\beta}\sum_{k,n} \mathrm{tr}(J^l_i G(\mathbf{k}, i(\omega_n+\nu_m)) J_j G(\mathbf{k}, i\omega_n)) = \\ &= -\frac{2}{V}\sum_k \frac{n^F(E_+) - n^F(E_-)}{\lambda((i\nu_m)^2 - (2\lambda)^2)} \times \\ &\times \{\nu_m \frac{\partial\varepsilon}{\partial k_i}\epsilon_{lns}\lambda_n\frac{\partial\lambda_s}{\partial k_j} + 2\lambda\frac{\partial\varepsilon}{\partial k_i}(\lambda^l\frac{\partial\lambda}{\partial k_j} - \lambda\frac{\partial\lambda_l}{\partial k_j})\} \end{aligned} \quad (4)$$

In a marked difference from the dielectric function in Eq.[3], the spin conductivity above depends on the kinetic energy $\varepsilon(k)$ through the term $\frac{\partial\varepsilon}{\partial k_i}$. The structure of the the two terms in Eq.[3] and Eq.[4] is fundamentally different for generic $\lambda_i(k)$.

For the Rashba Hamiltonian the summation transforms in an integral over the 2-dimensional k-space and $\lambda_x = \alpha k_y,\ \ \lambda_y = -\alpha k_x,\ \ \lambda_z = 0,\ \ \lambda = \alpha k$ and hence the first (reactive) term (which contains the fully antisymmetric ϵ_{abc} symbol) in Eq.[3] vanishes and we get:

$$\begin{aligned} \epsilon_{ij} &= -\frac{8\pi e^2}{\hbar\omega^2}\int \frac{d^2k}{(2\pi)^2}\frac{n(E_+) - n(E_-)}{\omega^2 - (2\lambda)^2}\lambda(\frac{\partial\lambda}{\partial k_i}\frac{\partial\lambda}{\partial k_j} - \frac{\partial\lambda_a}{\partial k_i}\frac{\partial\lambda_a}{\partial k_j}) = \\ &= \delta_{ij}\frac{2e^2\alpha^3}{\hbar\omega^2}\int_{k_+}^{k_-}\frac{k^2 dk}{(2\alpha k)^2 - \omega^2} \end{aligned} \quad (5)$$

where $k_\pm$ are the Fermi momenta of the $E_\pm$ bands. The above expression is in agreement with [1]. The spin conductance in Eq.[4] sees the last term vanish and gives:

$$\begin{aligned} \sigma^l_{ij}(\omega) &= \frac{Q^l_{ij}(\omega)}{-i\omega} = \\ &= -\frac{2e}{V}\sum_k \frac{n(E_+) - n(E_-)}{\lambda(\omega^2 - (2\lambda)^2)}\cdot\frac{\partial\varepsilon}{\partial k_i}\epsilon_{lnm}\lambda_n\frac{\partial\lambda_m}{\partial k_j} \end{aligned} \quad (6)$$

The only nonzero value is for $l = 3$ (spin polarized out of plane with in-plane electric field):

$$\sigma^3_{12} = \frac{e\alpha}{2\pi m}\int_{k_+}^{k_-}\frac{k^2 dk}{(2\alpha k)^2 - \omega^2} \quad (7)$$

As noticed before [1], the spin-orbit part of the dielectric function and the spin conductivity in Eq.[5] and Eq.[7] have the same frequency dependence. However, this is due to λ_i being linear in the momentum components k_j. This is the case in pure Rashba and Dresselhauss systems, as well as for a generic linear combination of the two. For λ linear in k, both factors $\frac{\partial\varepsilon}{\partial k_i}\epsilon_{lns}\lambda_n\frac{\partial\lambda_s}{\partial k_j}$ from the spin conductance in Eq[4] and $\lambda^2(\frac{\partial\lambda}{\partial k_i}\frac{\partial\lambda}{\partial k_j} - \frac{\partial\lambda_a}{\partial k_i}\frac{\partial\lambda_a}{\partial k_j})$ from the polarizability in Eq[3] are quadratic in k since $\partial\lambda_i/\partial k_j$ is a constant and ε, as kinetic energy, is quadratic in k. A generic λ would give different, spectrum-specific dependence.

The magnetic susceptibility in the Rashba model exhibits similar behavior. For generic spin-orbit coupling λ we have:

$$\begin{aligned} \chi_{ij}(i\nu_m) &= -\frac{1}{V}\int_0^\beta \langle T\sigma_i(u)\sigma_j\rangle e^{i\nu_m u} du = \\ &= \frac{1}{V\beta}\sum_{k,n}\mathrm{tr}(\sigma_i G(\mathbf{k}, i(\omega_n+\nu_m))\sigma_j G(\mathbf{k}, i\omega_n)) = \\ &= -\frac{2}{V}\sum_k \frac{n^F(E_+) - n^F(E_-)}{\lambda[(i\nu_m)^2 - (2\lambda)^2]}[\nu_m\epsilon_{ijk}\lambda_k + 2(\lambda^2\delta_{ij} - \lambda_i\lambda_j)] \end{aligned} \quad (8)$$

and we see that this differs in tensor structure from both the spin conductance and the polarizability, both of which contain derivatives of the spin-orbit coupling. However, for the Rashba Hamiltonian we obtain:

$$\chi_{ij}(\omega) = -\frac{\alpha}{\pi}\int_{k_+}^{k_-}\frac{k^2 dk}{(2\alpha k)^2 - \omega^2} \quad (9)$$

which has the same integral dependence as both the polarizability and the spin-conductance. This is again because λ is linear in k. However, a relation between the spin current and the magnetic susceptibility, (but not them being identical, as in the Rashba case), should exist in the generic case on general grounds. In spin 1/2 systems spin orbit coupling can always be thought of as

a fictitious k-dependent, internal magnetic field (the spin orbit coupling term is always first order in the spin operator $\lambda_i(k)\sigma_i$ since for spin 1/2 products of spin operators can be always expressed as linear combination of the 3 Pauli matrices). The presence of an electric field gives an non-zero expectation value for the momentum $< k >$ and hence for the fictitious internal magnetic field $< \lambda(k) >$. This creates a uniform magnetization proportional and the coefficient of proportionality is χ. However, since spin is not conserved in spin 1/2 systems, it will precess around the internal magnetic field $\lambda(k)$. This gives rise to an extra-term in the continuity equations which turns out to be proportional to the spin current, as in [2]. For the Rashba case, the two effects are related, as shown in [2] and this relates the spin conductance to the susceptibility, as also shown above.

We now turn our attention to the Luttinger Hamiltonian for spin $S = 3/2$ holes in the valence band of centrosymmetric cubic semiconductors:

$$H_L = \frac{1}{2m}(\gamma_1 + \frac{5}{2}\gamma_2)k^2 - \frac{\gamma_2}{m}(\mathbf{k}\cdot\mathbf{S})^2 \tag{10}$$

It can be cast in a form similar to the Rashba case by the transformation [7]:

$$\Gamma^1 = \frac{2}{\sqrt{3}}\{S_y, S_z\},\ \ \Gamma^2 = \frac{2}{\sqrt{3}}\{S_z, S_x\},\ \ \Gamma^3 = \frac{2}{\sqrt{3}}\{S_y, S_x\}$$
$$\Gamma^4 = \frac{1}{\sqrt{3}}(S_x^2 - S_y^2),\ \ \Gamma^5 = S_z^2 - \frac{5}{4}I_{4\times4}, \tag{11}$$

which satisfy the $SO(5)$ Clifford algebra $\Gamma^a\Gamma^b + \Gamma^b\Gamma^a = 2\delta_{ab}I_{4\times4}$. The Hamiltonian becomes:

$$H_L = \varepsilon(k) + d_a\Gamma^a,\ \ \varepsilon(k) = \frac{\gamma_1}{2m}k^2,\ \ a = 1, ..., 5 \tag{12}$$

where

$$d_1 = -\sqrt{3}\frac{\gamma_2}{m}k_zk_y,\ d_2 = -\sqrt{3}\frac{\gamma_2}{m}k_xk_z,\ d_3 = -\sqrt{3}\frac{\gamma_2}{m}k_xk_y,$$
$$d_4 = -\frac{\sqrt{3}}{2}\frac{\gamma_2}{m}(k_x^2 - k_y^2),\ d_5 = -\frac{1}{2}\frac{\gamma_2}{m}(2k_z^2 - k_x^2 - k_y^2) \tag{13}$$

where γ_1, γ_2 are material dependent parameters, and m is the electron mass. Since it is a time-invariant parity even fermionic hamiltonian, the band structure is composed of two doubly degenerate bands called light and heavy hole bands corresponding to helicity $\pm1/2$ and $\pm3/2$ with energies $E_\pm = \varepsilon(k) \pm d(k)$, $d^2 = d_ad_a$. The Green's function reads:

$$G(\mathbf{k}, i\omega) = \frac{1}{i\omega + \mu - H_L} = \frac{i\omega + \mu - \varepsilon(k) + d_a(k)\Gamma_a}{(i\omega + \mu - \varepsilon(k))^2 - d^2(k)} \tag{14}$$

Introducing as before the current operator $J_i = \frac{\partial H_L}{\partial k_i}$ we have the following expression for the spin-orbit coupling part of the dielectric function:

$$\epsilon_{ij}(\omega) = \frac{4\pi i\sigma_{ij}(\omega)}{\omega}$$

$$\sigma_{ij}(\omega) = \frac{e^2}{2\hbar}\frac{Q_{ij}(\omega)}{-i\omega}$$

$$Q_{ij}(i\nu_m) = -\frac{1}{V}\int_0^\beta < TJ_i(u)J_j > e^{i\nu_m u}du =$$
$$= -\frac{8}{V}\sum_k \frac{n^F(E_+) - n^F(E_-)}{(i\nu_m)^2 - (2d)^2}d(\frac{\partial d}{\partial k_i}\frac{\partial d}{\partial k_j} - \frac{\partial d_a}{\partial k_i}\frac{\partial d_a}{\partial k_j})] \tag{15}$$

Using the identity $\frac{\partial d_a}{\partial k_i}\frac{\partial d_a}{\partial k_j} = (\frac{\gamma_2}{m})^2(k_ik_j + 3k^2\delta_{ij})$, as well as doing the integrals over the angles we get

$$\epsilon_{ij}(\omega) = \delta_{ij}\frac{16e^2\gamma_2^3}{\pi\hbar m^3\omega^2}\int_{k_+}^{k_-}\frac{k^6dk}{(2\frac{\gamma_2}{m}k^2)^2 - \omega^2} \tag{16}$$

The spin conductivity is obtained as the response of the spin current to an applied electric field. A straightforward but tedious calculation gives:

$$\sigma_{ij}^l(\omega) = e\frac{Q_{ij}^l(\omega)}{-i\omega}$$

$$Q_{ij}^l(i\nu_m) = -\frac{4\nu_m}{V}\sum_k \frac{n^F(E_+) - n^F(E_-)}{d(i\nu_m)^2 - (2d)^2}\times$$
$$\times\ \eta_{ab}^l\left[2d_b\frac{\partial d_a}{\partial k_j}\frac{\partial\varepsilon}{\partial k_i} + \epsilon_{abcde}d_e\frac{\partial d_c}{\partial k_i}\frac{\partial d_d}{\partial k_j}\right] \tag{17}$$

where $Q_{ij}^l(i\nu_m) = -\frac{1}{V}\int_0^\beta\langle TJ_i^l(u)J_j\rangle e^{i\nu_m u}du$ and where η_{ab}^l, $l = 1, 2, 3$, $a, b = 1, ..., 5$ is a tensor antisymmetric in a, b relating the spin-3/2 matrix S^l to the $SO(5)$ generators $\Gamma^{ab} = \frac{1}{2i}[\Gamma^a, \Gamma^b]$, $S^l = \eta_{ab}^l\Gamma^{ab}$. The explicit form of η_{ab}^l is [7]:

$$\eta_{15}^1 = \frac{\sqrt{3}}{4},\ \ \eta_{23}^1 = -\frac{1}{4},\ \ \eta_{14}^1 = \frac{1}{4},\ \ \eta_{25}^2 = -\frac{\sqrt{3}}{4},$$

$$\eta_{13}^2 = \frac{1}{4},\ \ \eta_{24}^2 = \frac{1}{4},\ \ \eta_{34}^3 = -\frac{1}{2},\ \ \eta_{12}^3 = -\frac{1}{4} \tag{18}$$

From Eq.[15] and Eq.[20] we can see that the spin current and dielectric function again have very different structure. This is reflected when we introduce the explicit k dependence:

$$\sigma_{ij}^k(\omega) = \epsilon_{ijk}\frac{e\gamma_2}{\pi^2 m^2}(\gamma_1 + \frac{2\gamma_2}{3})\int_{k_+}^{k_-}\frac{k^4 dk}{(2\frac{\gamma_2}{m}k^2)^2 - \omega^2} \quad (19)$$

From Eq.[16] and Eq.[19] we see that they have different spectra. The result for spin conductivity in this note differs from the result in [7] because we use the full spin operator while [7] use a conserved spin operator, which commutes with the Hamiltonian. Different from the case of the Rashba Hamiltonian, here we can define a conserved spin current by projecting the spin-3/2 operator into the light and heavy-hole bands [7]. The expression for spin conductivity in this case takes an extremely nice, topological form and can be obtained in our case by neglecting the first term in the square bracket of Eq.[20] or by neglecting the γ_1 term in Eq.[19]. The conserved spin exists in the Luttinger Hamiltonian due to the presence of degenerate sub-bands, and it is rigourous only in inversion symmetric semiconductors where the bands are doubly degenerate. Different from the spin 1/2 case (Rashba, Dresselhauss, etc.) where the spin is not conserved and precesses around a fictitious internal magnetic field given by the spin-orbit coupling, the conserved spin will not precess and will satisfy a continuity equation. While in general spin is defined as the physical quantity which couples to a magnetic field, in this case the spin-3/2 in the valence band of cubic semiconductors is actually a combination of both angular spin-1 and Pauli spin 1/2 degrees of freedom. Hence it is not immediately obvious whether the spin or the conserved spin is the meaningful physical quantity which would couple to an external field. Depending on whether the gap between the heavy and light hole states at the Fermi energy is larger than thermal and disorder energies or not it might be the case that either the conserved or the non-conserved spin is the physically measurable quantity.

We now turn to the calculation of magnetic susceptibility in the Luttinger model. We start with the usual, non-conserved spin and find:

$$\begin{aligned}\chi_{ij}(i\nu_m) &= -\frac{1}{V}\int_0^\beta < TS_i(u)S_j > e^{i\nu_m u}du = \\ &= \frac{4}{V}\delta_{ij}\sum_k \frac{n^F(E_+) - n^F(E_-)}{d(i\nu_m)^2 - (2d)^2}\eta^i_{ab}\eta^j_{bc}d_a d_c \\ &= \frac{4}{V}\sum_k \frac{n^F(E_+) - n^F(E_-)}{(i\nu_m)^2 - (2d)^2}d = \\ &= -\delta_{ij}\frac{2}{\pi^2}\frac{\gamma_2}{m}\int_{k_+}^{k_-}\frac{k^4 dk}{(2\frac{\gamma_2}{m}k^2)^2 - \omega^2} \quad (20)\end{aligned}$$

where $\omega = i\nu_m$. We see that this is has the same functional form (frequency dependence) as the spin conductance, thought not the same tensor structure. Since we have computed the magnetic susceptibility of the usual, non-conserved spin, this is expected, by arguments similar to the spin 1/2 case, since the non-conserve spin also precesses. However, when we compute the spin conductance of the conserved spin $S_i^{cons} = P_{HH}S_iP_{HH} + P_{LH}S_iP_{LH}$ where P_{HH} and P_{LH} are the projection operators in the heavy and light holes, we obtain:

$$\chi_{ij}(i\nu_m) = -\frac{1}{V}\int_0^\beta < TS_i^{cons}(u)S_j^{cons} > e^{i\nu_m u}du = 0 \quad (21)$$

The magnetic susceptibility vanishes whereas the spin conductance is finite.

We now attempt to make some general statements about spin currents in spin-orbit coupling systems. First consider spin-1/2 coupling systems given by the generic Hamiltonian in Eq.[1]. Since they have no degenerate states, in these systems there is no conserved spin. If the spin-orbit coupling is linear in k ($\lambda \sim k$) then the spin conductivity is 'universal' in the sense that it does not depend on the spin-orbit coupling strength. This is valid for both 2 and 3 dimensions, as long as the Fermi energy is positive. The universal spin conductance is an artifact of the spin-orbit coupling being linear in the momentum.

For spin 3/2 systems with time-reversal and parity invariance one can define a conserved spin current with finite response to an electric field by projecting the spin onto the degenerate states. The nice topological form obtained in [7] is entirely due to the fact that the system is spin-3/2. Spin 3/2 systems do not have universal conductance as the the Hamiltonian is quadratic and not linear in k.

The spin current conductivities computed here are valid for the disorder-free case. The introduction of disorder in the Rashba case gives the remarkable result that the vertex correction completely cancels the disorder-free spin current [8]. This does not happen in the Luttinger case due to the inversion symmetry of the Luttinger Hamiltonian. Moreover tedious calculations reveal that vertex correction does not cancel the spin conductivity in a variety of other Hamiltonians, including k^3 Dresselhauss and Rashba plus k-linear Dresselhauss spin splitting. In this sense, it seems that the cancelation in the Rashba case is accidental and not due to a symmetry (Ward identity) of the system.

In the present short note we have computed the dielectric function, the spin conductivity and the magnetic susceptibility for both Rashba and Luttinger type Hamiltonians in a way which makes their gauge structure clear, and showed that, in general, they have different frequency dependencies. We also discussed some generic issues about spin-orbit coupling systems.

We wish to thank S.C. Zhang for countless discussions on the subject of spin-orbit coupling and spin currents. Support was offered through the Stanford SGF program as well as by the NSF under grant numbers

DMR-0342832 and the US Department of Energy, Office of Basic Energy Sciences under contract DE-AC03-76SF00515.

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